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EXCELLENCE PROGRAM - SYJC (SCIENCE), 2019-2020
SYNOPSIS
MATHEMATICS & STATISTICS – PART 2
PROBABILITY DISTRIBUTION [3 MARKS FOR H.S.C.]

❖ **Random Variable :**

A random variable (r, v) is a function whose domain is a sample space of random experiment and whose co-domain is the set of real numbers. i.e. $X : S \rightarrow R$.

❖ **Types of Random Variables:**

(i) **Discrete Random Variable :**

A random variable which can take countable number of isolated values either finite or infinite is called as a discrete random variable. (Values may be positive or negative)

(ii) **Continuous Random Variable :**

A random variable which can take any value in a given interval is called as a continuous random variable.

❖ **Probability Mass function or Discrete Probability Distribution :**

If X is a discrete r.v. taking values x_1, x_2, \dots, x_n then with each x_i we assign a number

$p_i = P[X = x_i], i = 1, 2, \dots, n$ called as probability of x_i such that,

(i) $0 \leq p_i \leq 1, i = 1, 2, \dots, n$

(ii)
$$\sum_{i=1}^n P[X = x_i] = \sum_{i=1}^n p_i = 1$$

Such a function P is called as probability mass function (p.m.f.) of X . Set of ordered pairs $(x_i, p_i), i = 1, 2, \dots, n$ is called as probability distribution of discrete r.v. X .

❖ **Cumulative Distribution Function (c.d.f.):**

Cumulative distribution function (c.d.f.) of a discrete r.v. X at some fixed value is defined as

$$F(x) = P[X \leq x], x \in R.$$

$$= \int_{-\infty}^x f(x) dx$$

c.d.f. is also called as Distribution function (d.f.)

Also, $P(X > x) = 1 - P[X \leq x] = 1 - F(x)$.

❖ **Expected value, Variance and Standard Deviation :**

If X is a discrete r.v. taking values x_1, x_2, \dots, x_n with respective probabilities p_1, \dots, p_n , then

- (i) Mean or Expected value of X , denoted by μ or $E(X)$ is defined as

$$\mu = E(X) = \sum_{i=1}^n x_i p_i.$$

$E(X)$ is also called as mathematical expectation.

- (ii) Variance of X denoted by σ^2 or $\text{Var}(X)$ is given by

$$\sigma^2 = \text{Var}(X)$$

$$= \sum_{i=1}^n x_i^2 p_i - \left(\sum_{i=1}^n x_i p_i \right)^2$$

$$= E(X^2) - [E(X)]^2.$$

- (iii) Standard Deviation of X , denoted by σ_x is given by

$$\sigma_x = \sqrt{\text{Var}(X)}.$$

❖ **Probability Density Function (p.d.f.) :**

A real valued function $f(x)$ is called as a Probability density function (p.d.f) of a continuous r.v. X if

- (i) $f(x) \geq 0$, $\forall x \in R$ and

(ii) $\int_{-\infty}^{\infty} f(x) dx = 1$

It takes values in the interval (a, b) , then

$$f(x) \geq 0 \text{ for } a < x < b$$

and $\int_a^b f(x) dx = 1$

$$\text{Also, } P(a \leq x \leq b) = P(a < x \leq b) = P(a \leq x < b) = \int_a^b f(x) dx$$

$$\text{and } P[X = a] = 0 = P[X = b].$$